

NAG Toolbox for MATLAB

f07ag

1 Purpose

f07ag estimates the condition number of a real matrix A , where A has been factorized by f07ad.

2 Syntax

```
[rcond, info] = f07ag(norm_p, a, anorm, 'n', n)
```

3 Description

f07ag estimates the condition number of a real matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – string

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **a(lda,*)** – double array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The LU factorization of A , as returned by f07ad.

3: **anorm** – double scalar

If **norm_p** = '1' or 'O', the 1-norm of the **original** matrix A .

If **norm_p** = 'I', the ∞ -norm of the **original** matrix A .

anorm must be computed either **before** calling f07ad or else from a **copy** of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **a**.

n, the order of the matrix *A*.

Constraint: $n \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, iwork

5.4 Output Parameters

1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of *A*. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, *A* is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter *i* had an illegal value on entry. The parameters are numbered as follows:

1: **norm_p**, 2: **n**, 3: **a**, 4: **lda**, 5: **anorm**, 6: **rcond**, 7: **work**, 8: **iwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07ag involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2n^2$ floating-point operations but takes considerably longer than a call to f07ae with one right-hand side, because extra care is taken to avoid overflow when *A* is approximately singular.

The complex analogue of this function is f07au.

9 Example

```
norm_p = '1';
a = [5.25, -2.95, -0.95, -3.8;
     0.3428571428571429, 3.891428571428571, 2.375714285714285,
     0.4128571428571428;
     0.300952380952381, -0.4631179637787567, -1.513859275575135,
```

```
0.2948206069505628;  
      -0.2114285714285714, -0.3298825256975037,  0.004723367663983707,  
0.1313732394878517];  
anorm = 9.74;  
[rcond, info] = f07ag(norm_p, a, anorm)  
  
rcond =  
      0.0066  
info =  
      0
```
